Exercise 2.1

Question 1:

If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$, find the values of x and y.

Solution 1:

It is given that $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore, $\frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$

$\frac{x}{3}+1=\frac{5}{3}$

$\Rightarrow\frac{x}{3}=\frac{5}{3}-1\quad y-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow\frac{x}{3}=\frac{2}{3}\quad\Rightarrow y=\frac{1}{3}+\frac{2}{3}$

$\Rightarrow x=2\quad\Rightarrow y=1$

$\therefore x=2$ and $y=1$

Question 2:

If the set A has 3 elements and the set B $=\{3,4,5\}$, then find the number of elements in $(A\times B)$?

Solution 2:

It is given that set A has 3 elements and the elements of set B are 3, 4, and 5.

$\Rightarrow$ Number of elements in set B = 3

Number of elements in $(A\times B)$

= (Number of elements in A) × (Number of elements in B)

$=3\times3=9$

Thus, the number of elements in $(A\times B)$ in 9.

Question 3:

If G = {7, 8} and H = {5, 4, 2}, find $G\times H$ and $H\times G$.

Solution 3:

G = $\{7,8\}$ and H $=\{5,4,2\}$

We know that the Cartesian product $P\times Q$ of two non-empty sets P and Q is defined as

$P\times Q-\{(p,q): p\in P, q\in Q\}$

$\therefore G\times H=\{(7,5),(7,4),(7,2),(8,5),(8,4),(8,2)\}$

$H\times G=\{(5,7),(5,8),(4,7),(4,8),(2,7),(2,8)\}$

Question 4:
State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.
(i) If P = {m, n} and Q = {n, m}, then $P \times Q = \{(m,n),(n,m)\}$.
(ii) If A and B are non-empty sets, then $A \times B$ is a non-empty set of ordered pairs (x, y) such that $x \in A$ and $y \in B$.
(iii) If $A = \{1,2\}, B = \{3,4\}$, then $A \times \{B \cap \varnothing\} = \varnothing$.

Solution 4:
(i) False
If $P = \{m,n\}$ and $Q = \{n,m\}$, then
$P \times Q = \{(m,m),(m,n),(n,m),(n,n)\}$
(ii) True
(iii) True

Question 5:
If $A = \{-1,1\}$, find $A \times A \times A$.

Solution 5:
If is known that for any non-empty set $A, A \times A \times A$ is defined as $A \times A \times A = \{(a,b,c) : a,b,c \in A\}$
It is given that $A = \{-1,1\}$

$$\therefore A \times A \times A = \begin{Bmatrix} (-1,-1,-1),(-1,-1,1),(-1,1,-1),(-1,1,1), \\ (1,-1,-1),(1,-1,1),(1,1,-1),(1,1,1) \end{Bmatrix}$$

Question 6:
If $A \times B = \{(a,x),(a,y),(b,x),(b,y)\}$. Find A and B.

Solution 6:
If is given that $A \times B = \{(a,x),(a,y),(b,x),(b,y)\}$
We know that the Cartesian product of two non-empty sets P and Q is defined as $P \times Q = \{(p,q) : p \in P, q \in Q\}$
$\therefore$ A is the set of all first elements and B is the set of all second elements.
Thus, $A = \{a,b\}$ and $B = \{x,y\}$

Question 7:
Let $A = \{1,2\}, B = \{1,2,3,4\}, C = \{5,6\}$ and $D = \{5,6,7,8\}$. Verify that

(i) $A\times(B\cap C)=(A\times B)\cap(A\times C)$

(ii) $A\times C$ is a subset of $B\times D$

Solution 7:

(i) To verify: $A\times(B\cap C)=(A\times B)\cap(A\times C)$

We have $B\cap C=\{1,2,3,4\}\cap\{5,6\}=\varnothing$

$\therefore L.H.S.=A\times(B\cap C)=A\times\varnothing=\varnothing$

$A\times B=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4)\}$

$A\times C=\{(1,5),(1,6),(2,5),(2,6)\}$

$\therefore R.H.S.=(A\times B)\cap(A\times C)=\varnothing$

$\therefore L.H.S.=R.H.S.$

Hence, $A\times(B\cap C)=(A\times B)\cap(A\times C)$

(ii) To verify: $A\times C$ is a subset of $B\times D$

$A\times C=\{(1,5),(1,6),(2,5),(2,6)\}$

$A\times D=\{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),(3,7),$
$(3,8),(4,5),(4,6),(4,7),(4,8)\}$

We can observe that all the elements of set $A\times C$ are the elements of set $B\times D$. Therefore, $A\times C$ is a subset of $B\times D$.

Question 8:

Let $A=\{1,2\}$ and $B=\{3,4\}$. Write $A\times B$. How many subsets will $A\times B$ have? List them.

Solution 8:

$A=\{1,2\}$ and $B=\{3,4\}$

$\therefore A\times B=\{(1,3),(1,4),(2,3),(2,4)\}$

$\Rightarrow n(A\times B)=4$

We know that if C is a set with $n(C)=m$, then $n[P(C)]=2^m$.

Therefore, the set $A\times B$ has $2^4=16$ subsets. These are

$\varnothing,\{(1,3)\},\{(1,4)\},\{(2,3)\},\{(2,4)\},\{(1,3)(1,4)\},\{(1,3),(2,3)\},$

$\{(1,3),(2,4)\},\{(1,4),(2,3)\},\{(1,4)(2,4)\},\{(2,3)(2,4)\}$

$\{(1,3),(1,4),(2,3)\},\{(1,3),(1,4),(2,4)\},\{(1,3),(2,3),(2,4)\}$

$\{(1,4),(2,3),(2,4)\},\{(1,3),(1,4),(2,3),(2,4)\}$

Question 9:

Let A and B be two sets such that $n(A)=3$ and $n(B)=2$. If $(x,1),(y,2),(z,1)$ are in $A\times B$, find A and B, where x,y and z are distinct elements.

Solution 9:

It is given that $n(A)=3$ and $n(B)=2$; and $(x,1),(y,2),(z,1)$ are in $A\times B$.

We know that

A = Set of first elements of the ordered pair elements of $A\times B$

B = Set of second elements of the ordered pair elements of $A\times B$.

$\therefore x, y,$ and z are the elements of A; and 1 and 2 are the elements of B.

Since $n(A)=3$ and $n(B)=2$,

It is clear that $A=\{x,y,z\}$ and $B=\{1,2\}$.

Question 10:

The Cartesian product $A\times A$ has 9 elements among which are found $(-1,0)$ and (0, 1). Find the set A and the remaining elements of $A\times A$.

Solution 10:

We know that if $n(A)=p$ and $n(B)=q$, then $n(A\times B)=pq$.

$\therefore n(A\times A)=n(A)\times n(A)$

It is given that $n(A\times A)=9$

$\therefore\ n(A)\times n(A)=9$

$\Rightarrow n(A)=3$

The ordered pairs $(-1,0)$ and (0, 1) are two of the nine elements of $A\times A$.

We know that $A\times A=\{(a,a):a\in A\}$. Therefore, $-1, 0,$ and 1 are elements of A.

Since $n(A)=3$, it is clear that $A=\{-1,0,1\}$.

The remaining elements of set $A\times A$ are $(-1,-1),(-1,1),(0,-1),(0,0),(1,-1),(1,0)$, and $(1,1)$.

Exercise 2.2

Question 1:

Let $A=\{1,2,3...14\}$. Define a relation R from A to A by $R=\{(x,y):3x-y=0\}$, where $x,y\in A$. Write down its domain, codomain and range.

Solution 1:

The relation R from A to A is given as $R=\{(x,y):3x-y=0, \text{where}\, x,y\in A\}$

i.e., $R=\{(x,y):3x=y, \text{where}\, x,y\in A\}$

$\therefore R=\{(1,3),(2,6),(3,9),(4,12)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.

$\therefore$ Domain of $R=\{1,2,3,4\}$

The whole set A is he codomain of the relation R.

$\therefore$ Codomain of $R=A=\{1,2,3....14\}$

The range of R is the set of all second elements of the ordered pairs in the relation.

$\therefore$ Range of $R=\{3,6,9,12\}$

Question 2:

Define a relation R on the set **N** of natural numbers by $R=\{(x,y):y=x+5, x\,\text{is a natural number less than}\, 4; x,y\in \mathbf{N}\}$. Depict this relationship using roster form. Write down the domain and the range.

Solution 2:

$R=\{(x,y):y=x+5,\ x\,\text{is a natural number less than}\, 4, x,y\in\mathbf{N}\}$

The natural numbers less than 4 are 1, 2, and 3.

$\therefore R=\{(1,6),(2,7),(3,8)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.

$\therefore$ Domain of $R=\{1,2,3\}$

The range of R is the set of all second elements of the ordered pairs in the relation.

$\therefore$ Range of $R=\{6,7,8\}$

Question 3:

$A=\{1,2,3,5\}$ and $B=\{4,6,9\}$. Define a relation R from A to B by $R=\{(x,y):\text{the difference between}\, x\,\text{and}\, y\,\text{is odd}; x\in A, y\in B\}$. Write R in roster form.

Solution 3:

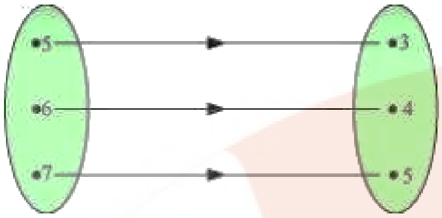
$A=\{1,2,3,5\}$ and $B=\{4,6,9\}$

$R=\{(x,y):\text{the difference between}\, x\,\text{and}\, y\,\text{is odd}; x\in A, y\in B\}$

$\therefore R=\{(1,4),(1,6),(2,9),(3,4),(3,6),(5,4),(5,6)\}$

Question 4:

The given figure shows a relationship between the sets P and Q. Write this relation
(i) in set-builder form
(ii) in roster form.
What is its domain and range?



Solution 4:

According to the given figure, $P=\{5,6,7\}, Q=\{3,4,5\}$

(i) $R=\{(x,y): y=x-2; x\in P\}$ or $R=\{(x,y): y=x-2 \text{ for } x=5,6,7\}$

(ii) $R=\{(5,3),(6,4),(7,5)\}$

Domain of $R=\{5,6,7\}$

Range of $R=\{3,4,5\}$

Question 5:

Let $A=\{1,2,3,4,6\}$. Let R be the relation on A defined by $\{(a,b): a,b\in A, b \text{ is exactly divisible by a}\}$.

(i) Write R in roster form
(ii) Find the domain of R
(iii) Find the range of R.

Solution 5:

$A=\{1,2,3,4,6\}, R=\{(a,b): a,b\in A, b \text{ is exactly divisible by } a\}$

(i) $R=\{(1,1),(1,2),(1,3),(1,4),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(6,6)\}$

(ii) Domain of $R=\{1,2,3,4,6\}$

(iii) Range of $R=\{1,2,3,4,6\}$

Question 6:

Determine the domain and range of the relation R defined by $R=\{(x,x+5): x\in\{0,1,2,3,4,5\}\}$.

Solution 6:

$R=\{(x,x+5): x\in\{0,1,2,3,4,5\}\}$

$\therefore R=\{(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)\}$

$\therefore$ Domain of $R=\{0,1,2,3,4,5\}$

Range of $R=\{5,6,7,8,9,10\}$

Question 7:

Write the relation $R=\{(x,x^3): x \text{ is a prime number less than } 10\}$ in roster form.

Solution 7:

$R=\{(x,x^3): x \text{ is a prime number less than } 10\}$. The prime numbers less than 10 are 2, 3, 5 and 7.

$\therefore R=\{(2,8),(3,27),(5,125),(7,343)\}$

Question 8:

Let $A=\{x,y,z\}$ and $B=\{1,2\}$. Find the number of relations from A to B.

Solution 8:

It is given that $A=\{x,y,z\}$ and $B=\{1,2\}$.

$\therefore A\times B=\{(x,1),(x,2),(y,1),(y,2),(z,1),(z,2)\}$

Since $n(A\times B)=6$, the number of subsets of $A\times B$ is 2^6.

Therefore, the number of relations from A to B is 2^6.

Question 9:

Let R be the relation on **Z** defined by $R=\{(a,b): a,b\in \mathbf{Z},\ a-b \text{ is an integer}\}$. Find the domain and range of R.

Solution 9:

$R=\{(a,b): a,b\in \mathbf{Z},\ a-b \text{ is an integer}\}$

It is known that the difference between any two integers is always an integer.

$\therefore$ Domain of R = **Z**

Range of R = **Z**

Exercise 2.3

Question 1:
Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$

(ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

(iii) $\{(1,3),(1,5),(2,5)\}$

Solution 1:
$\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$

Since 2, 5, 8, 11, 14 and 17 are the elements of the domain of the given relation having their unique images, this relation is a function. Here, domain $=\{2,5,8,11,14,17\}$ and range $=\{1\}$

(ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

Since 2, 4, 6, 8, 10, 12 and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain $=\{2,4,6,8,10,12,14\}$ and range $=\{1,2,3,4,5,6,7\}$

(iii) $\{(1,3),(1,5),(2,5)\}$

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

Question 2:
Find the domain and range of the following real function:

(i) $f(x)=-|x|$ (ii) $f(x)=\sqrt{9-x^2}$

Solution 2:
(i) $f(x)=-|x|,\ x\in R$

We know that $|x|=\begin{cases} x, & if\ x\geq 0 \\ -x & if\ x<0 \end{cases}$

$\therefore f(x)=-|x|=\begin{cases} -x, & if\ x\geq 0 \\ x, & if\ x<0 \end{cases}$

Since f(x) is defined for $x\in \mathbf{R}$, the domain of f is $\mathbf{R}$.

It can be observed that the range of $f(x)=-|x|$ is all real numbers except positive real numbers.

$\therefore$ The range of f is $(-\infty,0]$.

(ii) $f(x)=\sqrt{9-x^2}$

Since $\sqrt{9-x^2}$ is defined for all real numbers that are greater than or equal to -3 and less than or equal to 3, the domain of $f(x)$ is $\{x:-3\leq x\leq 3\}$ or $[-3,3]$.

For any value of x such that $-3\leq x\leq 3$, the value of $f(x)$ will lie between 0 and 3.

$\therefore$ The range of $f(x)$ is $\{x : 0 \le x \le 3\}$ or $[0,3]$.

Question 3:

A function f is defined by $f(x) = 2x - 5$.

(i) $f(0)$, (ii) $f(7)$ (iii) $f(-3)$

Solution 3:

The given function is $f(x) = 2x - 5$

Therefore,

(i) $f(0) = 2\times 0 - 5 = 0 - 5 = -5$

(ii) $f(7) = 2\times 7 - 5 = 14 - 5 = 9$

(iii) $f(-3) = 2\times(-3) - 5 = -6 - 5 = -11$

Question 4:

The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$. Find

(i) t (0) (ii) $t(28)$ (iii) $t(-10)$

(iv) The value of C, when $t(C) = 212$

Solution 4:

The given function is $t(C) = \frac{9C}{5} + 32$.

Therefore,

(i) $t(0) = \frac{9\times 0}{5} + 32 = 0 + 32 = 32$

(ii) $t(28) = \frac{9\times 28}{5} + 32 = \frac{252+160}{5} = \frac{412}{5} = 82.4$

(iii) $t(-10) = \frac{9\times(-10)}{5} + 32 = 9\times(-2) + 32 = -18 + 32 = 14$

(iv) It is given that $t(C) = 212$

$\therefore 212 = \frac{9C}{5} + 32$

$\Rightarrow \frac{9C}{5} = 212 - 32$

$\Rightarrow \frac{9C}{5} = 180$

$\Rightarrow 9C = 180\times 5$

$\Rightarrow C = \frac{180\times 5}{9} = 100$

Thus, the value of t, when $t(C) = 212$, is 100.

Question 5:

Find the range of each of the following functions.

(i) $f(x)=2-3x,\ x\in \mathrm{R}, x>0$.

(ii) $f(x)=x^2+2, x,$ is a real number.

(iii) $f(x)=x, x$ is a real number.

Solution 5:

(i) $f(x)=2-3x, x\in \mathrm{R}, x>0$

The values of $f(x)$ for various values of real numbers $x>0$ can be written in the tabular form as

x	0.01	0.1	0.9	1	2	2.5	4	5	…
f(x)	1.97	1.7	- 0.7	-1	- 4	- 5.5	- 10	-13	…

Thus, it can be clearly observed that the range of *f* is the set of all real numbers less than 2.
i.e., range of $f=(-\infty,2)$

Alter:

Let $x>0$

$\Rightarrow 3x>0$

$\Rightarrow 2-3x<2$

$\Rightarrow f(x)<2$

$\therefore$ Range of $f=(-\infty,2)$

(ii) $f(x)=x^2+2,\ x,$ is a real number

The values of $f(x)$ for various of real numbers x can be written in the tabular form as

x	**0**	**±0.3**	**±0.8**	**±1**	**±2**	**±3**	**...**
f(x)	2	2.09	2.64	3	6	11	.....

x	0	±0.3	±0.8	±1	±2	±3	…
f(x)	2	2.09	2.64	3	6	11	…

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2.
i.e., range of $f=[2,\infty)$

Alter:

Let x be any real number. Accordingly,

$x^2\geq 0$

$\Rightarrow x^2+2\geq 0+2$

$\Rightarrow x^2+2\geq 2$

$\Rightarrow f(x)\geq 2$

$\therefore$ Range of $f=[2,\infty)$

(iii) $f(x)=x, x$ is a real number

It is clear that the range of f is the set of all real numbers.

$\therefore$ Range of $f = \mathbf{R}$.

Miscellaneous Exercise

Vashu Panwar

Question 1:

The relation f is defined by $f(x)=\begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10 \end{cases}$

The relation g is defined by $g(x)=\begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$

Show that f is a function and g is not a function.

Solution 1:

The relation f is defined as

$$f(x)=\begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10 \end{cases}$$

It is observed that for

$0 \le x < 3, \quad f(x) = x^2$

$3 < x \le 10, \quad f(x) = 3x$

Also, at $x=3$, $f(x)=3^2=9$ or $f(x)=3\times 3=9$ i.e., at $x=3$, $f(x)=9$

Therefore, for $0 \le x \le 10$, the images of $f(x)$ are unique. Thus, the given relation is a function.

The relation g is defined as

$$g(x)=\begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$$

It can be observed that for $x=2$, $g(x)=2^2=4$ and $g(x)=3\times 2=6$

Hence, element 2 of the domain of the relation g corresponds to two different images i.e., 4 and 6.

Hence, this relation is not a function.

Question 2:

If $f(x)=x^2$, find $\dfrac{f(1.1)-f(1)}{(1.1-1)}$

Solution 2:

$f(x)=x^2$

$$\therefore \frac{f(1.1)-f(1)}{(1.1-1)} = \frac{(1.1)^2-(1)^2}{(1.1-1)} = \frac{1.21-1}{0.1} = \frac{0.21}{01} = 2.1$$

Question 3:

Find the domain of the function $f(x)=\dfrac{x^2+2x+1}{x^2-8x+12}$

Solution 3:

The given function is $f(x)=\dfrac{x^2+2x+1}{x^2-8x+12}$

$$f(x)=\frac{x^2+2x+1}{x^2-8x+12}=\frac{x^2+2x+1}{(x-6)(x-2)}$$

It can be seen that function f is defined for all real numbers except at x = 6 and x = 2. Hence, the domain of f is $\mathbf{R}-\{2,6\}$.

Question 4:

Find the domain and the range of the real function f defined by $f(x)=\sqrt{(x-1)}$

Solution 4:

The given real function is $f(x)=\sqrt{(x-1)}$

It can be seen that $\sqrt{(x-1)}$ is defined for $f(x)=x\geq 1$.

Therefore, the domain of f is the set of all real numbers greater than or equal to 1 i.e., the domain of $f=[1,\infty)$.

As $x\geq 1 \Rightarrow (x-1)\geq 0 \Rightarrow \sqrt{(x-1)}\geq 0$

Therefore, the range of f is the set of all real numbers greater than or equal to 0 i.e., the range of $f=[0,\infty)$.

Question 5:

Find the domain and the range of the real function f defined by $f(x)=|x-1|$.

Solution 5:

The given real function is $f(x)=|x-1|$.

It is clear that $|x-1|$ is defined for all real numbers.

$\therefore$ Domain of $f=\mathrm{R}$

Also, for $x\in \mathrm{R}=|x-1|$ assumes all real numbers.

Hence, the range of *f* is the set of all non-negative real numbers.

Question 6:

Let $f=\left\{\left(x,\dfrac{x^2}{1+x^2}\right): x\in \mathrm{R}\right\}$

be a function from **R** into **R**. Determine the range of *f*.

Solution 6:

$$f=\left\{\left(x,\frac{x^2}{1+x^2}\right):x\in\mathbf{R}\right\}$$

$$=\left\{(0,0),\left(\pm0.5,\frac{1}{5}\right),\left(\pm1,\frac{1}{2}\right),\left(\pm1.5,\frac{9}{13}\right),\left(\pm2,\frac{4}{5}\right),\left(3,\frac{9}{10}\right),\left(4,\frac{16}{17}\right),....\right\}$$

The range of f is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.
[Denominator is greater numerator]. Thus, range of f = [0, 1)

Question 7:

Let $f,g:\mathrm{R}\rightarrow\mathrm{R}$ be defined, respectively by $f(x)=x+1, g(x)=2x-3$. Find $f+g, f-g$ and $\frac{f}{g}$.

Solution 7:

$f,g:\mathrm{R}\rightarrow\mathrm{R}$ is defined as $f(x)=x+1,\ g(x)=2x-3$

$(f+g)(x)=f(x)+g(x)=(x+1)+(2x-3)=3x-2$

$\therefore(f+g)(x)=3x-2$

$(f-g)(x)=f(x)-g(x)=(x+1)-(2x-3)=x+1-2x+3=-x+4$

$\therefore(f-g)(x)=-x+4$

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)},g(x)\neq0,x\in\mathrm{R}$

$\therefore\left(\frac{f}{g}\right)(x)=\frac{x+1}{2x-3},2x-3\neq0$ or $2x\neq3$

$\therefore\left(\frac{f}{g}\right)(x)=\frac{x+1}{2x-3},x\neq\frac{3}{2}$

Question 8:

Let $f=\{(1,1),(2,3),(0,-1),(-1,-3)\}$ be a function from **Z** to **Z** defined by $f(x)=ax+b$, for some integers a,b. Determine a,b.

Solution 8:

$f=\{(1,1),(2,3),(0,-1),(-1,-3)\}$ and $f(x)=ax+b$

$(1,1)\in f\Rightarrow f(1)=1\Rightarrow a\times1+b=1$

$\Rightarrow a+b=1$

$(0,-1)\in f\Rightarrow f(0)=-1\Rightarrow a\times0+b=-1$

On substituting $b=-1$ in $a+b=1$

We obtain $a+(-1)=1 \Rightarrow a=1+1=2$. Thus, the respective values of a and b are 2 and -1.

Question 9:

Let R be a relation from **N** to **N** defined by $R=\{(a,b): a,b \in \mathbf{N} \text{ and } a=b^2\}$. Are the following true?

(i) $(a,a) \in R$, for all $a \in \mathbf{N}$

(ii) $(a,b) \in R$, implies $(b,a) \in R$

(iii) $(a,b) \in R, (b,c) \in R$ implies $(a,c) \in R$.

Justify your answer in each case.

Solution 9:

$R=\{(a,b): a,b \in \mathbf{N} \text{ and } a=b^2\}$

(i) It can be seen that $2 \in \mathbf{N}$; however, $2 \neq 2^2 = 4$.

Therefore, the statement "$(a,a) \in R$, for all $a \in \mathbf{N}$" is not true.

(ii) It can be seen that $(9,3) \in \mathbf{N}$ because $9,3 \in \mathbf{N}$ and $9=3^2$. Now, $3 \neq 9^2 = 81$; therefore, $(3,9) \notin \mathbf{N}$

Therefore, the statement "$(a,b) \in R$, implies "$(b,a) \in R$" is not true.

(iii) It can be seen that $(9,3) \in R, (16,4) \in R$ because $9,3,16,4 \in \mathbf{N}$ and $9=3^2$ and $16=4^2$. Now, $9 \neq 4^2 = 16$; therefore, $(9,4) \notin \mathbf{N}$

Therefore, the statement "$(a,b) \in R, (b,c) \in R$ implies $(a,c) \in R$" is not true.

Question 10:

Let $A=\{1,2,3,4\}, B=\{1,5,9,11,15,16\}$ and $f=\{(1,5),(2,9),(3,1),(4,5),(2,11)\}$. Are the following true?

(i) f is a relation from A to B

(ii) f is a function from A to B

Justify your answer in each case.

Solution 10:

$A=\{1,2,3,4\}$ and $B=\{1,5,9,11,15,16\}$

$$\therefore A \times B = \{(1,1),(1,5),(1,9),(1,11),(1,15),(1,16),(2,1),(2,5),(2,9),(2,11),(2,15),(2,16)$$
$$(3,1),(3,5),(3,9),(3,11),(3,15),(3,16),(4,1),(4,5),(4,9),(4,11),(4,15),(4,16)\}$$

It is given that $f=\{(1,5),(2,9),(3,1),(4,5),(2,11)\}$

(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product $A \times B$.

Thus, f is a relation from A to B.

(ii) Since the same first element i.e, 2 corresponds to two different images i.e., 9 and 11, relation f is not a function.

Question 11:

Let f be the subset of $\mathbf{Z}\times\mathbf{Z}$ defined by $f=\{(ab, a+b): a, b\in\mathbf{Z}\}$. If f a function from **Z** to **Z**: Justify your answer.

Solution 11:

The relation *f* is defined as $f=\{(ab, a+b): a, b\in\mathbf{Z}\}$

We know that a relation *f* from a set A to a set B is said to be a function if every element of set A has unique images in set B.

Since $(2, 6, -2, -6\in Z, (2\times 6, 2+6), (-2\times -6, -2+-6))\in f$ i.e., $(12, 8), (12, -8)\in f$

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and -8. Thus, relation *f* is not a function.

Question 12:

Let $A=\{9, 10, 11, 12, 13\}$ and let $f: A\to\mathbf{N}$ be defined by $f(n)=$ the highest prime factor of *n*. Find the range of *f*.

Solution 12:

$A=\{9, 10, 11, 12, 13\}$ and let $f: A\to\mathbf{N}$ is defined as $f(n)=$ The highest prime factor of *n*

Prime factor of 9 = 3
Prime factors of 10 = 2, 5
Prime factor of 11 = 11
Prime factor of 12 = 2, 3
Prime factor of 13 = 13

$\therefore f(9)=$ The highest prime factor of 9 = 3

$f(10)=$ The highest prime factor of 10 = 5

$f(11)=$ The highest prime factor of 11 = 11

$f(12)=$ The highest prime factor 12 = 3

$f(13)=$ The highest prime factor of 13 = 13

The range of *f* is the set of all $f(n)$, where $n\in A$.

$\therefore$ Range of $f=\{3, 5, 11, 13\}$
